EE3123 Tutorial 1 (Solution)
AC Power Circuits and Components I

Name: **Student No.:**

Q1

Convert the following instantaneous currents to phasors, using $\cos(\omega t)$ as the reference. Give your answers in both rectangular and polar form.

(a) $i(t) = 500\sqrt{2}\cos(\omega t - 30)$

(b) $i(t) = 4\sin(\omega t + 30)$

(c) $i(t) = 5\cos(\omega t - 15) + 4\sqrt{2}\sin(\omega t + 30)$

Solution

(a) $\bar{I} = 500\angle -30° = 433.01 - j250$

(b) $i(t) = 4\sin(\omega t + 30°) = 4\cos(\omega t + 30° - 90°) = 4\cos(\omega t - 60°)$

$\bar{I} = 2.83\angle -60° = 1.42 - j2.45$

(c) $\bar{I} = \left(5/\sqrt{2}\right)\angle -15° + 4\angle -60° = (3.42 - j0.92) + (2 - j3.46)$

$= 5.42 - j4.38 = 6.964\angle -38.94°$

Q2

A 60-Hz, single-phase source with $V = 277\angle 30°$ volts is applied to a circuit element. (a) Determine the instantaneous source voltage. Also determine the phasor and instantaneous currents entering the positive terminal if the circuit element is (b) a 20-Ω resistor, (c) a 10-mH inductor, and (d) a capacitor with 25-Ω reactance.

Solution

(a) $\upsilon(t) = 277\sqrt{2}\cos(\omega t + 30°) = 391.7\cos(\omega t + 30°)\,\text{V}$

(b) $\bar{I} = \bar{V}/20 = 13.85\angle 30°\,\text{A}$

$i(t) = 19.58\cos(\omega t + 30°)\,\text{A}$

(c) $\bar{Z} = j\omega L = j(2\pi 60)(10\times 10^{-3}) = 3.771\angle 90°\,\Omega$

$\bar{I} = \bar{V}/\bar{Z} = (277\angle 30°)/(3.771\angle 90°) = 73.46\angle -60°\,\text{A}$

$i(t) = 73.46\sqrt{2}\cos(\omega t - 60°) = 103.9\cos(\omega t - 60°)\,\text{A}$

(d) $\bar{Z} = -j25\,\Omega$

$\bar{I} = \bar{V}/\bar{Z} = (277\angle 30°)/(25\angle -90°) = 11.08\angle 120°\,\text{A}$

$i(t) = 11.08\sqrt{2}\cos(\omega t + 120°) = 15.67\cos(\omega t + 120°)\,\text{A}$

Q3

The instantaneous voltage across a circuit element is $v(t)$=400cos(ωt+30°) volts, and the instantaneous current entering the positive terminal of the circuit element is $i(t)$=100sin(ωt +10°) A. Calculate

(a) the instantaneous power absorbed,

(b) the real power (state whether it is delivered or absorbed),

(c) the reactive power (state whether delivered or absorbed),

(d) the power factor (state whether lagging or leading).

[Note: By convention the power factor cos(δ-β) is positive. If |δ-β| is greater than 90°, then the reference direction for current may be reversed, resulting in a positive value of cos(δ-β)].

Solution

(a) $p(t) = v(t)i(t) = \left[400\cos(\omega t + 30°)\right]\left[100\cos(\omega t - 80°)\right]$

$$= \frac{1}{2}(400)(100)\left[\cos 110° + \cos(2\omega t - 50°)\right]$$

$$= -6840.4 + 2\times 10^4 \cos(2\omega t - 50°)\ \text{W}$$

(b) $P = VI\cos(\delta - \beta) = (282.84)(70.71)\cos(30° + 80°)$

$= -6840\ \text{W}$ Delivered

$= +6840\ \text{W}$ Absorbed

(c) $Q = VI\sin(\delta - \beta) = (282.84)(70.71)\sin 110°$

$= 18.79\ \text{kVAR}$ Absorbed

(d) The phasor current $(-\bar{I}) = 70.71\angle -80° + 180° = 70.71\angle 100°$ A leaves the positive terminal of the generator.

The generator power factor is then $\cos(30° - 100°) = 0.3420$ leading

Q4

A 60-Hz, single-phase source with $V=277\underline{/30°}$ volts is applied to a circuit element. Determine the instantaneous power, real power, and reactive power absorbed by

(a) the 20-Ω resistor,

(b) the 10-mH inductor,

(c) the capacitor with 25-Ω reactance.

Also determine the source power factor and state whether lagging or leading.

Solution

(a) $p(t) = \upsilon(t)i(t) = 391.7 \times 19.58 \cos^2(\omega t + 30°)$

$$= 0.7669 \times 10^4 \left(\frac{1}{2}\right)\left[1 + \cos(2\omega t + 60°)\right]$$

$$= 3.834 \times 10^3 + 3.834 \times 10^3 \cos(2\omega t + 60°)\,\mathrm{W}$$

$$P = VI\cos(\delta - \beta) = 277 \times 13.85 \cos 0° = 3.836\,\mathrm{kW}$$

$$Q = VI\sin(\delta - \beta) = 0\,\mathrm{VAR}$$

$$\text{Source Power Factor} = \cos(\delta - \beta) = \cos(30° - 30°) = 1.0$$

(b) $p(t) = \upsilon(t)i(t) = 391.7 \times 103.9 \cos(\omega t + 30°)\cos(\omega t - 60°)$

$$= 4.07 \times 10^4 \left(\frac{1}{2}\right)\left[\cos 90° + \cos(2\omega t - 30°)\right]$$

$$= 2.035 \times 10^4 \cos(2\omega t - 30°)\ \mathrm{W}$$

$$P = VI\cos(\delta - \beta) = 277 \times 73.46 \cos(30° + 60°) = 0\,\mathrm{W}$$

$$Q = VI\sin(\delta - \beta) = 277 \times 73.46 \sin 90° = 20.35\,\mathrm{kVAR}$$

$$pf = \cos(\delta - \beta) = 0 \text{ Lagging}$$

(c) $p(t) = \upsilon(t)i(t) = 391.7 \times 15.67 \cos(\omega t + 30°)\cos(\omega t + 120°)$

$$= 6.138 \times 10^3 \left(\frac{1}{2}\right)\left[\cos(-90°) + \cos(2\omega t + 150°)\right] = 3.069 \times 10^3 \cos(2\omega t + 150°)\,\mathrm{W}$$

$$P = VI\cos(\delta - \beta) = 277 \times 11.08 \cos(30° - 120°) = 0\,\mathrm{W}$$

$$Q = VI\sin(\delta - \beta) = 277 \times 11.08 \sin(-90°)$$

$$= -3.069\,\mathrm{kVAR} \quad \text{Delivered}$$

$$pf = \cos(\delta - \beta) = \cos(-90°) = 0 \text{ Leading}$$

Q5

Consider a single-phase load with an applied voltage $v(t)$=150 cos(ωt+10°) volts and load current $i(t)$=5cos(ωt-50°) A.

(a) Determine the power triangle.

(b) Find the power factor and specify whether it is lagging or leading.

(c) Calculate the reactive power supplied by capacitors in parallel with the load that correct the power factor to 0.9 lagging.

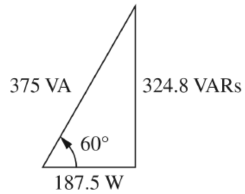
Solution

(a) $\overline{S} = \overline{V}\,\overline{I}^* = \left(\dfrac{150}{\sqrt{2}}\angle 10°\right)\left(\dfrac{5}{\sqrt{2}}\angle -50°\right)^* = 375\angle 60°$

$= 187.5 + j324.8$

$P = \text{Re}\,\overline{S} = 187.5\ \text{W Absorbed}$

$Q = \text{Im}\,\overline{S} = 324.8\ \text{VARS Absorbed}$



(b) $pf = \cos(60°) = 0.5$ Lagging

(c) $Q_S = P\tan Q_S = 187.5\tan\left[\cos^{-1}0.9\right] = 90.81\ \text{VARS}$

$Q_C = Q_L - Q_S = 324.8 - 90.81 = 234\ \text{VARS}$

Q6

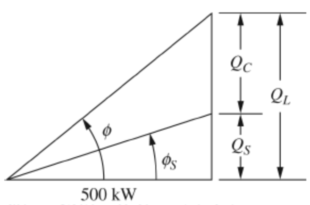
An industrial plant consisting primarily of induction motor loads absorbs 500 kW at 0.6 power factor lagging.

(a) Compute the required kVA rating of a shunt capacitor to improve the power factor to 0.9 lagging.

(b) Calculate the resulting power factor if a synchronous motor rated at 500 hp with 90% efficiency operating at rated load and at unity power factor is added to the plant instead of the capacitor. Assume constant voltage (1 hp = 0.746 kW).

Solution

(a)



$Q_L = P\tan\phi_L = 500\tan 53.13° = 666.7\ \text{kVAR}$

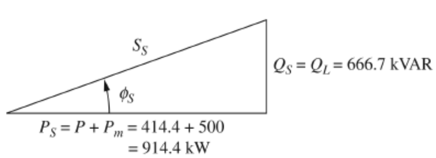
$\phi_S = \cos^{-1}0.9 = 25.84°$

$Q_S = P\tan\phi_S = 500\tan 25.84° = 242.2\ \text{kVAR}$

$Q_C = Q_L - Q_S = 666.7 - 242.2 = 424.5\ \text{kVAR}$

$S_C = Q_C = 424.5\ \text{kVA}$

(b) The Synchronous motor absorbs $P_m = \dfrac{(500)0.746}{0.9} = 414.4\ \text{kW}$ and $Q_m = 0\ \text{kVAR}$



Source $\text{PF} = \cos\left[\tan^{-1}(666.7/914.4)\right] = 0.808$ Lagging